Gauging the Shadow Sector with SO(3)

R.J. Lindebaum, G.B. Tupper and R.D. Viollier

Institute of Theoretical Physics and Astrophysics,
Department of Physics, University of Cape Town, Rondebosch 7701,
South Africa.

e-mail: viollier@physci.uct.ac.za

Abstract

We examine the phenomenology of a low-energy extension of the Standard Model, based on the gauge group $SU(3) \otimes SU(2) \otimes U(1) \otimes SO(3)$, with $SO(3)$ operating in the shadow sector. This model offers $\nu_e \to \nu_s$ and $\nu_\mu \to \nu_\tau$ oscillations as the solution of the solar and atmospheric neutrino problems. Moreover, it provides a neutral heavy shadow lepton X that could play the role of a cold dark matter particle.

With the accumulated evidence for neutrino oscillations [1] comes the challenge of understanding the origin of neutrino mass. Since the simple Higgs triplet [2] is ruled out by LEP [3], most approaches centre on singlet fermions and some variant of the see-saw mechanism [4]. The question then arises how these new particles fit into the larger theory.

One of the most novel proposals [5], inspired by $E_8 \otimes E_8^{'}$ superstring theory, is that fermions, which are light because they are non-singlets under a low-energy "shadow gauge group" $G^{'}$, could play an important role in this regard. The implementation of this idea, however, involves several specific assumptions: $G^{'}$ is isomorphic to the Standard Model $SU(3) \otimes SU(2) \otimes U(1)$, with matter fields in the fundamental representation coming in three generations. None of these inputs are compulsory. In fact, in the superstring scenario, compactification yields E_6 as the grand unified gauge group and it is also responsible for the generation structure, while the $E_8^{'}$ is left intact and can break down to $G^{'}$ in many ways. Furthermore, even if one can identify $SU(2)^{'} \times U(1)^{'}$ as a subgroup of $G^{'}$, there are still many possible matter representations [6] including neutral heavy shadow leptons which could mix with the active neutrinos.

It is therefore important to explore other candidates for the low-energy group $G^{'}$, in order to obtain a clearer picture on this issue. Thus, in this paper, we will examine $G' = SO(3)$, the Georgi-Glashow model [7], which being vector like, is anomaly free. We assume no generation replication and, since there is no information on the "charged" shadow leptons, we dispense with the artifice of putting in a mass by hand. The shadow leptons appear as triplets

$$\vec{\psi}_L = \begin{pmatrix} E^+ \\ X \\ e^{'-} \end{pmatrix}_L \quad , \quad \vec{\psi}_R = \begin{pmatrix} E^+ \\ \nu^{'} \\ e^{'-} \end{pmatrix}_R \tag{1}$$

and the singlet $S_R = X_R$. At energies much below some large scale Λ, a connection through the charge-neutral sector is phenomenologically afforded by

$$-\mathcal{L}_{G-G'} = \frac{f_{L\ell}}{\Lambda}\, \vec{\psi}_L^T \cdot \vec{\phi}\, C\, H_2^\dagger\, L_\ell + \frac{f_{R\ell}}{\Lambda}\, \bar{L}_\ell\, H_2\, \vec{\phi} \cdot \vec{\psi}_R + \text{h.c.}\ , \tag{2}$$

where L_ℓ and H_2 are the usual lepton doublets and u-like Higgs field, respectively, $\vec{\phi}$ is the shadow Higgs triplet and $\ell = e, \mu, \tau$ a generation index. Omitted in (2) is X_R, since by the survival hypothesis [8], it is expected to pick up a large Majorana mass.

At low energies with $< H_2 >= v/\sqrt{2}$, $< \phi >= v'$ and

$$\vec{m}_{L/R} = \frac{vv'}{\sqrt{2}\Lambda} \left(f_{L/R\,e}\ ,\ f_{L/R\,\mu}\ ,\ f_{L/R\,\tau}\right)^T \ , \tag{3}$$

we obtain, in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu'^c, X_L)$, the neutral mass matrix

$$\underset{\sim}{\mathcal{M}} \ = \ \begin{pmatrix} \underset{\sim}{O} & \vec{m}_R & \vec{m}_L \\ \vec{m}_R^T & 0 & 0 \\ \vec{m}_L^T & 0 & M \end{pmatrix} \ , \tag{4}$$

where M is the Majorana mass of X_L resulting from the shadow see-saw mechanism. This matrix has rank 1, the normalized zero eigenvector being

$$V^0 \ = \ \frac{1}{|\sin\ \chi|} \begin{pmatrix} \hat{m}_L \ \times \ \hat{m}_R \\ 0 \\ 0 \end{pmatrix} \ , \tag{5}$$

and χ is the angle between $\hat{m}_L$ and $\hat{m}_R$. Taking $M \gg |\vec{m}_{L/R}|, \cos\chi \ll 1$, and denoting $x \equiv |\vec{m}_L|/m_X$, the remaining eigenvalues and eigenvectors are

$$-\, m_3 \approx -\vec{m}_L^2/m_X \ , \ \ V^3 \simeq \frac{1}{|\sin\chi|\sqrt{1 + x^2 \sin^2\chi}} \begin{pmatrix} \hat{m}_R \times \hat{m}_L \times \hat{m}_R \\ 0 \\ -x\ \sin^2\ \chi \end{pmatrix} \tag{6}$$

$$\pm\, m_{1,2} \ \simeq \ \pm\, |\vec{m}_R| \ + \ \frac{m_3}{2}\ \cos^2\ \chi, \qquad V^{1,2} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \pm\ \hat{m}_R \\ 1 \\ 0 \end{pmatrix} \tag{7}$$

$$m_4 \ \simeq \ M \ + \ \frac{\vec{m}_L^2}{M} \ \equiv \ m_X, \qquad V^4 \ \simeq \frac{1}{\sqrt{1 + x^2}} \begin{pmatrix} x\ \hat{m}_L \\ 0 \\ 1 \end{pmatrix} \ . \tag{8}$$

Note that $|\Delta\, m_{12}| = |\Delta\, m_{03}| \cos^2\, \chi$.

Since all oscillation solutions of the solar neutrino problem involve smaller $|\Delta m|$ than the atmospheric neutrino problem, we take $\hat{m}_R = (1,\, 0,\, 0)$, i.e. $\nu_e \to \nu_s$ with maximal mixing, which is consistent with both the solar neutrino data and the nucleosynthesis bound [9]. As the atmospheric neutrino data are consistent with maximal mixing $\nu_\mu \to \nu_\tau$, we take $\hat{m}_L = (\cos\chi, \sin\chi/\sqrt{2}, \sin\chi/\sqrt{2})$, so that $\Delta m^2_{\rm atm} \approx 3 \times 10^{-3}$ eV^2 fixes $m_3 \approx 5.5 \times 10^{-2}$ eV. The vacuum solution $\Delta m^2_\odot \approx 6.5 \times 10^{-11}$ eV^2 is consistent with $\chi \approx 90^o$.

Next we turn to cosmology in the presence of the neutral heavy lepton X. The decay $X \to \nu'\gamma'$, where γ' is the shadow photon, is absent because, e'^- and E^+ being degenerate, their contributions [6] cancel. The $\nu' 2\gamma'$ mode is allowed and can be estimated using the results of [10]. Taking $m_E \approx v'$ in the shadow see-saw mechanism, we arrive at

$$\tau_{X \to \nu' 2\gamma'} \approx 10^{18} \left(\frac{\rm keV}{m_X}\right)^5 \left(\frac{\Lambda}{v}\frac{e}{e'}\right)^4 \ {\rm yr}\ . \tag{9}$$

A far more stringent constraint follows from the fact that $X \to 3\nu$ occurs via the weak neutral current for which a simple calculation yields

$$\tau_{X \to 3\nu} = 1.7 \times 10^{16} \left(\frac{\rm keV}{m_X}\right)^4 \ {\rm yr}\ . \tag{10}$$

Thus a neutral heavy shadow lepton of mass $m_X < 36$ keV is stable on the lifetime of the universe, and it can serve as dark matter if it is much less abundant than ordinary neutrinos, so that it is consistent with the cosmological bounds.

In order to avoid gross conflicts, in particular the production of $SO(3)$ monopoles in the early universe [11], it is assumed that the Big Bang explodes asymmetrically into the visible sector, with the shadow sector remaining cold or empty [12]. Under this condition, sterile neutrals such as X may only be produced through oscillations [13] or gravitational interaction. If there is a large lepton number asymmetry suppressing oscillations, these neutral leptons will be produced with $\Omega \approx 1$ and behave like cold dark matter, for m_X of the order of 10 keV [14].

Even under conservative assumptions [15], the X particles may form compact objects of a size $R \geq$ (100 km s$^{-1}/\sigma$) AU, where σ is the velocity dispersion. However, as has recently been shown the fermions will undergo a first-order gravitational phase transition [16], forming degenerate fermion stars as they cool. The latent heat may be disposed of by ejecting some of the fermions or gravitational cooling [17].

There is circumstantial astrophysical evidence for the existence of such a neutral fermion in the mass range of 10 to 20 keV. In fact, modelling the violent compact dark object at the centre of M87 [18] with mass $M = (3.2 \pm 0.9) \times 10^9\ M_\odot$, as a degenerate fermion star near the Oppenheimer–Volkoff limit, constrains [19] the fermion mass to

$$12.4 \text{ keV} \ \leq\ m_X \ \leq\ 16.5 \text{ keV} \ . \tag{11}$$

Such a compact dark object would have a radius of 4.45 Schwarzschild radii. Thus there is little difference between a supermassive black hole and a fermion star of the same mass, at the Oppenheimer-Volkoff limit, as the last stable orbit around a black hole is 3 Schwarzschild radii anyway.

Similarly, modelling SgrA* near the centre of our galaxy [20] with mass $M = (2.6 \pm 0.2) \times 10^6$ M$_\odot$ as a fermion star, we obtain a lower bound for the fermion mass of $m_X >$ 15.9 keV, from the observed motion of stars near SgrA* [21] which constrains the radius of the fermion star to less than 0.018 pc. The enigmatic radio and infrared emission of this object, interpreted in terms of standard thin disk accretion theory, gives us an upper limit for the fermion mass $m_X <$ 18 keV [22] from the drop of the emission spectrum at infrared wavelengths. Such a fermion star would differ very much from a supermassive black hole of the same mass, as the escape velocity from the fermion star would be only about $v_\infty \approx 1700$ km/s. Virtually all supermassive compact dark objects that have been observed so far at the centres of galaxies have masses in the range of $10^{6.5}$ to $10^{9.5} M_\odot$.

Fixing $m_X = 16$ keV, we obtain $|\vec{m}_L| \approx 30$ eV, while $|\vec{m}_R|$ must be an order of magnitude smaller to allow ν_e and ν_s to serve as a hot dark matter component [23]. The scale appearing in eq.(2) as well as v' can be estimated by combining eqs.(3), (6a) and $m_E \approx v'$; taking $|\vec{f}_L| \approx 1$ we obtain

$$\Lambda \approx \frac{v^2}{2m_3} \approx 5.7 \times 10^{14} \text{ GeV} \tag{12}$$

$$\frac{v'}{v} \approx \sqrt{\frac{m_X}{2m_3}} \approx 380 \ \ . \tag{13}$$

While Λ is much below the Planck scale the identification of the $SO(3)$ as a low-energy subgroup of E_8' is still possible as the string scale may be brought down by large compact dimensions [24].

In summary, we have examined a model with a low-energy shadow gauge group $SO(3)$ connected to the Standard Model through their neutral sectors. This model is capable of describing all the existing neutrino oscillation data (except LSND), and it provides for cold dark matter in the form of a neutral heavy lepton X in the shadow sector. This particle could play an important role in understanding the supermassive compact dark objects at the centres of galaxies in terms of degenerate heavy lepton stars.

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